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4th Semester

Econometrics (Properties of estimators)

: Properties of estimators :

The main criteria for a good estimator obtained from a small sample are :- (1) unbiasedness (2) least-variance (3) efficiency (4) best, linear, unbiasedness (BLU) (5) least mean-square-error (MSE), (6) Sufficiency.

(1) Unbiased estimator

The bias of an estimator is defined as the difference between its expected value and the true parameter.

$$Bias = E(\hat{b}) - b$$

An estimator is unbiased if its bias is zero, that is if $E(\hat{b}) = b$.

(2) Minimum - variance estimator (or best estimator)

An estimate is best when it has the smallest variance as compared with any other estimate obtained from other econometric methods. Symbolically $\hat{b}$ is best if

$$E[\hat{b} - E(\hat{b})]^2 < E[\tilde{b} - E(\tilde{b})]^2$$

or, $var(\hat{b}) < var(\tilde{b})$

where $\tilde{b}$ is any other (not necessarily unbiased) estimate of the true parameter b.

(3) Efficient estimator

An estimator is efficient when it possesses both the previous properties, that is it is unbiased and has the minimum variance as compared with any other unbiased estimator. Symbolically $\hat{b}$ is efficient if the following two conditions are fulfilled.

(a) $E(\hat{b}) = b$

and (b) $E[\hat{b} - E(\hat{b})]^2 < E[b^* - E(b^*)]^2$

where b^* is another unbiased estimate of the true b. In other words the efficient estimator is the minimum variance (best) estimator within the class of all unbiased estimators.

(4) Best, linear, unbiased estimator (BLUE)

Linear estimator: - An estimator is linear if it is a linear function of the sample observations; that is, if it is determined by a linear combination of the sample data. Given the sample observations $Y_1, Y_2, ——, Y_n$, a linear estimator will have the form

$$K_1 Y_1 + K_2 Y_2 + —— + K_n Y_n$$

where, the K_i's are some constants.

Best, linear, unbiased estimator (BLUE)

An estimator $\hat{b}$ is BLU if it is linear, unbiased and has the smallest variance as compared with all other linear unbiased estimators of the true b.

(5) Minimum mean-square-error (MSE) estimator

The mean-square-error criterion is a combination of the unbiasedness and the minimum variance properties. An estimator is a minimum MSE estimator if it has the smallest mean-square-error defined as the expected value of the squared differences of the estimator around the true population parameter

$$MSE(\hat{b}) = E[\hat{b} - b]^2$$

It can be shown that the MSE is equal to the variance of the estimator plus the square of its bias.

$$MSE(\hat{b}) = var(\hat{b}) + bias^2(\hat{b})$$

[Proof :-

$$MSE = E(\hat{b} - b)^2$$

$$= E\{[\hat{b} - E(\hat{b}) + E(\hat{b}) - b]\}^2$$

$$= E[\hat{b} - E(\hat{b})]^2 + [E(\hat{b}) - b]^2 + 2E\{[\hat{b} - E(\hat{b})][E(\hat{b}) - b]\}$$

But,

$$E[\hat{b} - E(\hat{b})]^2 = var(\hat{b})$$

$$[E(\hat{b}) - b]^2 = bias^2(\hat{b})$$

and $E\{[\hat{b} - E(\hat{b})][E(\hat{b}) - b]\} = 0$ because

$$E\{[\hat{b} - E(\hat{b})][E(\hat{b}) - b]\} = E\{\hat{b}E(\hat{b}) - [E(\hat{b})]^2 - \hat{b}b + bE(\hat{b})\}$$

$$= [E(\hat{b})]^2 - [E(\hat{b})]^2 - bE(\hat{b}) + bE(\hat{b}) = 0$$

Therefore, $MSE = var(\hat{b}) + bias^2(\hat{b})$]

(6) Sufficient estimator

A sufficient estimator is an estimator that utilises all the information a sample contains about the true parameter, it must use all the observations of the sample. This means that no other estimator can add any further information about the true population parameter which is being estimated.

: Large sample properties of Estimators :

When the sample is large these properties are assumed to hold only approximately. They are: (1) Asymptotic unbiasedness (2) Consistency (3) Asymptotic efficiency.

The concept of an asymptotic distribution means limiting distribution of the sequence of random variables