

Mugberia Gangadhar Mahavidyalaya
Department of Mathematics :: Class Text(2019)
Partial Differential Equations
Mathematics (Hons.): SEM-V: CT11: Full Marks 40

Any five from Group -A:

$2 \times 5 = 10$

1. Find the general solution of the PDE $uu_x + yu_y = x$.
2. Find the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$.
3. Find the order and degree of the PDE $p \tan y + q \tan x = \sec^2 z$.
4. Let $u(x, t)$, $x \in \mathfrak{R}$, $t \geq 0$ be the solution of the initial value problem $u_{xx} = u_{tt}$, $u(x, 0) = x$ and $u_t(x, 0) = 1$. Then find the value of $u(2, 2)$.
5. Let $a, b \in \mathfrak{R}$ be such that $a^2 + b^2 \neq 0$. Then verify that the Cauchy problem $au_x + bu_y = 1$, $x, y \in \mathfrak{R}$ with $u(x, y) = x$ on $ax + by = 1$ has a unique solution or not ?
6. The second order PDE $u_{yy} - yu_{xx} + x^3u = 0$ is **NET(MS): (June)2012**
(a) Elliptic for all $x \in \mathfrak{R}$, $y \in \mathfrak{R}$ (b) Parabolic for all $x \in \mathfrak{R}$, $y \in \mathfrak{R}$
(c) Elliptic for all $x \in \mathfrak{R}$, $y < 0$ (d) Hyperbolic for all $x \in \mathfrak{R}$, $y < 0$.
7. Find characteristic curve of the following PDEs :
(a) $yz \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = xy$
(b) $yz \frac{\partial z}{\partial x} + xz^2 \frac{\partial z}{\partial y} = xy$.

Any six questions from Group -B:

$5 \times 6 = 30$

1. Find the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0$, $z = 1$.
2. Find the equation of the integral surface of $x^2p + y^2q + z^2 = 0$ which passes through the hyperbola $xy = x + y$, $z = 1$
3. Find the equation of the integral surface satisfying $4yzp + q + 2y = 0$ and passing through the curve $y^2 + z^2 = 1$, $x + z = 2$ **IAS 1997**
4. Show that the equations $xp - yq = 0$, $z(xp + yq) = 2xy$ are compatible and solve them.

Ans. $z^2 = 2xy + k$ where k is a constant.

5. Reduce the following PDEs to canonical form : $x^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} - x \frac{\partial z}{\partial x} + 3y \frac{\partial z}{\partial y} - \frac{8y}{x} = 0$.
6. Find the solution of the equation $2z = p^2 + q^2 + 2(p - x)(q - y)$ which passes through the x-axis. **IAS 2002**
7. Find a complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ **IAS 1991**
8. Find the characteristics of the equation $p^2 + q^2 = 2$ and determine the integral surface which passes through the straight line $x = 0, z = y$.
9. Using the method of separation of variables solve

$$4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u, \quad \text{where } u(0, y) = 3e^{-y} - e^{-5y}$$