Mugberia Gangadhar Mahavidyalaya
Department of Mathematics
Internal Assessment Examination of B.Sc (Mathematics) SEM-II-2019
PAPER-CT4::Full Marks 20 :: Time -1 Hour

Any five questions from Group -A: $2 \times 5 = 10$

1. Let A be a 3×3 matrix with real entries. If three solutions of the linear system of differential equations $\dot{x}(t) = Ax(t)$ are given by $\begin{pmatrix} e^t - e^{2t} \\ -e^t + e^{2t} \\ e^t + e^{2t} \end{pmatrix}$, $\begin{pmatrix} -e^{2t} - e^{-t} \\ e^{2t} - e^{-t} \\ e^{2t} + e^{-t} \end{pmatrix}$ and $\begin{pmatrix} e^{-t} + 2e^t \\ e^{-t} - 2e^t \\ -e^{-t} + 2e^t \end{pmatrix}$. Then the sum of the diagonal entries of A is equal to $--$?
GATE(MA):2018

2. Let $(x(t), y(t))$ satisfy for $t > 0$
$\frac{dx}{dt} = -x + y, \frac{dy}{dt} = -y, \quad x(0) = y(0) = 1.$
Then find the value of $(x(t))$.

3. Solve : $\frac{dx}{y^2+yz+z^2} = \frac{dy}{z^2+zx+x^2} = \frac{dz}{x^2+xy+y^2}$.

4. Solve : $\frac{dx}{x^2+y^2} = \frac{dy}{2xy} = \frac{dz}{z(x+y)}$.

5. Solve : $\frac{dx}{x(x^2+3y^2)} = \frac{dy}{y(y^2+3x^2)} = \frac{dz}{2z(x^2+y^2)}$.

6. Consider the first order system of linear equations $\frac{dX}{dt} = AX$ where $A = \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix}$ and $X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$. Then **NET(MS): (Dec.)2011**
(a) the coefficient matrix A has a repeated eigenvalue $\lambda = 1$.

(b) there is only one linearly independent eigenvector $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(c) the general solution of the ODE is $(aX_1 - bX_2)e^t$, where a and b are arbitrary constants and $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $X_2 = \begin{pmatrix} t \\ \frac{1}{2} - t \end{pmatrix}$.
(d) the vectors X_1 and X_2 in the option (c) given above are linearly independent.

Any two questions from Group -B: $5 \times 2 = 10$

1. Solve: $(D^2+1)x + (D+1)y = t,\ 2x + (D+1)y = 0$, given that $x(0) = y(0) = 0$ and $Dx(0) = -5$.

2. Find the general solution and Fundamental Matrix for the system

$$\begin{aligned} \frac{dy_1}{dt} &= 3y_1 + y_2 \\ \frac{dy_2}{dt} &= -y_1 + y_2 \end{aligned}$$

3. Solve: $\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$ which contains the straight line $x+y=0,\ z=1$.