Mugberia Gangadhar Mahavidyalaya

Internal Assessment for M.Sc in Mathematics : Sem.-I(2018)

Paper-105:: Non-linear Dynamics

Answer any five: $2 \times 5 = 10$ :: Time 30 minutes

1. Find the steady state point for the system of equations $\dot{x} = x + 2y$ and $\dot{y} = x^2 + y$.

2. Determine the steady state and their stability of the differential equation $\dot{y} = f(y) = y^2 - 5y + 6$.

3. Discuss about the stability of the following system of differential equations: $\dot{x} = -x + y, \qquad \dot{y} = 4x - y$.

4. Investigate the stability of the zero solution of the system $\dot{x} = -y - x^3,\ \dot{y} = x - y^3$.

5. Prove that the zero solution of the system $\dot{x}_1 = -x_1 + x_2^2 + x_3^2,\ \dot{x}_2 = x_1 - 2x_2 + x_1^2,\ \dot{x}_3 = x_1 + 2x_2 - 3x_3 + x_2x_3$ is uniformly and asymptotically stable.

6. Prove that the zero solution of the system

$$\frac{d^3x}{dt^3} - 2\frac{d^2x}{dt^2} - \frac{dx}{dt} + 2x = \frac{d^2x}{dt^2}(x + \frac{dx}{dt})$$

   is unstable.

7. Show that all solution $\ddot{x} + \{a + c(1 + t^2)^{-1}\}x = e^{-t}$ are stable if $a > 0$.

8. By linearizing around the critical points, draw the phase plane portrait of $y'' + y - y^3 = 0$.