

M.Sc 4th Semester Internal Assessment Examination, 2020

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Operational Research Modeling-II)

Paper MTM – 405(Unit-I) ;; FULL MARKS 10

Time : 1 Hour

Answer Q. no. 1 and any one from the rest. 10

1. Answer any two questions: 4

(a) Find the curve $x=x(t)$ which minimize the functional $J = \int_0^1 (\dot{x}^2 + 1) dt$,

$$x(0) = 1 \quad \text{and} \quad x(1) = 2 \quad ?$$

(b) What is the failure rate? If the failure distribution Q has a density and

$$\text{failure rate } \lambda(t), \text{ show that } 1 - Q(t) = \exp\left[-\int_0^t \lambda(t) dt\right]$$

(c) Define entropy function and explain its importance.

(d) Draw the diagram of a communication system mentioning all the important components including noise system.

2. In a system, there are n number of components connected in series with reliability $R_i(t)=n, i=1, 2, \dots n$. Find reliability of the system.

$$\text{If } R_1(t) = R_2(t) = \dots = R_n(t) = e^{-\lambda t} \quad \text{then find the reliability of the system.} \quad 6$$

3. An electrochemical system is characterized by the ordinary differential equation

$$\frac{dx_1}{dt} = x_2 \quad \text{and} \quad \frac{dx_2}{dt} + x_2 = u \quad \text{where } u \text{ is the control variable chosen in such a way that the}$$

cost function $\frac{1}{2} \int_0^a (x_1^2 + 4u^2) dt$ is minimized. Show that if the boundary conditions

satisfied by the state variables are $x_1(0)=a, x_2(0)=b$, where a, b are constants and

$$x_1 \rightarrow 0, x_2 \rightarrow 0 \quad \text{as } t \rightarrow \infty, \text{ the optimal choice for } u \text{ is } u = -\frac{1}{2}x_1(t) + (1 - \sqrt{2})x_2(t). \quad 6$$

