Application of Derivatives

Maxima and Minima values of a function

Second Derivative Test :-

1. If $f'(c) > 0$ then $f(x)$ is increasing at $x = c$ and if $f'(c) < 0$ " $f(x)$ " decreasing at $x = c$
2. A function $f(x)$ has a maximum value at $x = c$ if $f'(c) = 0$, $f''(c) < 0$
3. A function $f(x)$ has a minimum value at $x = c$ if $f'(c) = 0$, $f''(c) > 0$
4. If $f'(c) = 0$, $f''(c) = 0$ and $f'''(c) \neq 0$ then $f(x)$ has neither a maximum nor a minimum at $x = c$, In this case $x = c$ is a point of inflexion.

Example

1. Find the value of x for which for which the function is increasing or decreasing where $f(x) = 3 + 4x - 4x^2$

Soln.

$$f(x) = 3 + 4x - x^2$$

$$f'(x) = 4 - 2x$$

i) If $f(x)$ is increasing function of x then we have

$f'(x) > 0$, or $4 - 2x > 0$ ~~or $2x > 4$ or $x > 2$~~

or $2x < 4$

or $x < 2$

ii) If $f(x)$ is decreasing function of x then we have

$f'(x) < 0$ or $4 - 2x < 0$

or $2x > 4$

or $x > 2$

2. Examine $f(x) = x^3 - 9x^2 + 24x - 12$ for maximum or minimum values.

Soln:

$$f(x) = x^3 - 9x^2 + 24x - 12$$

$$f'(x) = 3x^2 - 18x + 24$$

For maximum and minimum values of $f(x)$, we must have

$$f'(x) = 0$$

or

or $3x^2-18x+24=0$

or $3(x-2)(x-4)=0$

or $x=2, \text{or } 4.$

Again $f''(x)=6x-18$

At $x=2$, $f''(x)=6\cdot2-18=-6$ (–ve) or <0

Thus the function is maximum at $x=2$, the maximum value is

$$f(2)=2^3-9\cdot2^2+24\cdot2-12$$
$$=8-36+48-12$$
$$=8$$

At $x=4$, $f''(x)=6\cdot4-18=6$ (+ve), >0.

Thus the function is minimum at $x=4$, the minimum value is

$$f(4)=4^3-9\cdot4^2+24\cdot4-12$$
$$=4.$$

3. If $\frac{x}{2}+\frac{y}{3}=1$, find the minimum value of x^2+y^2

Soln:

$\frac{x}{2}+y/3=1$

or $3x+2y=6$

or $2y=6-3x$

or $y=\frac{6-3x}{2}$

Let $z=x^2+y^2=x^2+\left(\frac{6-3x}{2}\right)^2=x^2+\left(3-\frac{3}{2}x\right)^2$

$=x^2+9-9x+\frac{9}{4}x^2$

$\frac{dz}{dx}=2x+\frac{9}{2}x-9$

For maximum and minimum, we have

$\frac{dz}{dx}=0$

or $2x+\frac{9}{2}x-9=0$

or $13x=18$

or $x=\frac{18}{13}$

$$\frac{d^2z}{dx^2} = \frac{13}{2}$$

If $x = \frac{18}{13}$ $\frac{d^2z}{dx^2} = \frac{13}{2} > 0$

Hence, the function is minimum at $x = \frac{18}{13}$, the minimum value is

$$z = \left(\frac{18}{13}\right)^2 + \left(3 - \frac{3}{2}\times\frac{18}{13}\right)^2$$

$$= \frac{36}{13}.$$

4. Show that $y = x^3 - 8$ has neither a maximum nor a minimum value. Does the curve ~~has~~ have a point of inflexion?

<u>Soln.</u>

$$y = x^3 - 8$$

~~Now diff~~

$$\frac{dy}{dx} = 3x^2$$

$$\frac{d^2y}{dx^2} = 6x$$

For maximum and minimum value, we must have

$$\frac{dy}{dx} = 0$$

or $3x^2 = 0$

or $x^2 = 0$

or $x = 0$

If $x = 0$, $\frac{d^2y}{dx^2} = 6\cdot 0 = 0$

Hence the given function has neither maximum nor a minimum value

But $\frac{d^2y}{dx^2} = 0$, $\frac{d^3y}{dx^3} = 6 \neq 0$

So the given function has a point of inflexion at $x = 0$

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<u>N.B</u> :- If any problems arises, please contact $\left\{\begin{matrix} 8250625880 \\ \text{or } 9733842368 \end{matrix}\right\}$